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SPACETIMES WITH INTRINSIC SPHERICAL SYMMETRY

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When discussing spherically symmetric gravitational fields one usually starts from the assumption that the whole spacetime has the O(3) symmetry group. However, the standard justification for this assumption relies on measurements which are performed with the Newtonian concept of space in the background. The 4-dimensional geometry of the spacetime can be deduced from these measurements by indirect methods. It might be interesting, then, to see how much we can deduce from the weaker assumption that only the 3-dimensional hypersurfaces t = const are spherically symmetric, while the whole spacetime not necessarily shares this property. To be more precise, we assume that the spacetime can be foliated into a congruence of spacelike hypersurfaces which are spherically symmetric and are the level surfaces of a function which serves as a time coordinate in the spacetime.

For simplicity we shall also assume that:

1. The t-lines are orthogonal to the spherically symmetric hypersurfaces t = const.
2. The matter, if any is present, can move only in radial directions with respect to the center of symmetry of the 3-spaces. Any transversal motion could be easily detected, and so would constitute a too easy proof that 4-dimensional spherical symmetry is absent. It seems more interesting to deal with such models in which the lack of symmetry is cleverly masked.
3. The Einstein field equations are fulfilled, with the source being either a perfect fluid, or dust, or the Λ-term, or empty space.

The physical difference between the normal spherically symmetric spacetimes and the models considered here may be explained as follows. When speaking of the geometry in a 3-space t = const we forget about the possibility to measure the time flow. If the 3-spaces t = const are all spherically symmetric, the t-lines are orthogonal to these spaces, and, moreover, all the clocks placed upon one sphere go at the same rate, then the spacetime is spherically symmetric in the traditional sense [1]. The solution given below confirms this statement.

The work reported here is one of the possible specifications of the Collins' program [2] to investigate spacetimes with intrinsic symmetries. The calculations were performed with use of the symbolic formula-manipulation computer system ORTOCARTAN [3].

From the assumption 1 it follows that the metric, in a suitably chosen coordinate system, must be of the form:

$$ds^2 = D^2\,dt^2 - e^{2\mu}dr^2 - \delta^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2), \text{ where:}$$

$$D = D(t,r,\vartheta,\varphi),\quad \mu = \mu(t,r),\quad \delta = \delta(t,r).$$

The investigation of the field equations has given the following results [4]:

I. When δ = const, the source is necessarily the Λ-term, and only the solution of Nariai [5] is recovered, which is spherically symmetric in the traditional sense.

II. When $\delta \neq$ const, then either

IIa) δ cannot be chosen as the r-coordinate, and then only the solutions which are spherically symmetric in the traditional sense emerge, or

IIb) δ can be chosen as the r-coordinate, and then the new solution given below emerges as the most general metric which is concordant with our assumptions, but is not spherically symmetric in the traditional sense. Of course, the old well known solutions appear along with it.

The new solution has the form:

$$ds^2 = D^2\,dt^2 - (1 - Kr^2)^{-1}\,dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2) \qquad (1)$$

where:

$$D = r\left[A(t)\sin\vartheta\cos\varphi + B(t)\sin\vartheta\sin\varphi + C(t)\cos\vartheta\right] + E(t)(1 - Kr^2)^{1/2} + s$$

A, B, C, and E are arbitrary functions of t,
K is an arbitrary constant,
s = 0 or s = 1.

This metric is conformally flat. The density of energy ϵ and the pressure p are:

$$\varkappa\epsilon = -3K$$

$$\varkappa p = 3K(1 - 2s/3D).$$

Consequently, with $s = 0 \neq K$ the solution is one of constant curvature, and thus equivalent to the de Sitter solution, while with K = 0 the metric is flat, though in neither case it looks so at first sight.

With $s \neq 0 \neq K$ the solution is new and has no symmetries.

The functions A, B, C, and E determine the geodesic curvature of the t-lines. When A = = B = C = E = 0 the t-lines are geodesic, and when $A = B = C = \neq E$, $\Lambda \neq 0$ the principal curvature (i.e. acceleration) vector points in the radial direction (i.e. the t-lines deviate from geodesics only in the r-direction). In both cases the solution becomes spherically symmetric in the normal sense.

One may also investigate the intrinsically spatially homogeneous spacetimes in which the 3-spaces t = const are homogeneous, but the transitive group G is not the symmetry group of the whole spacetime. A preliminary study of such spacetimes was performed under the assumption that the spatial hypersurfaces are spherically symmetric in addition to being homogeneous, with the following results.

If the spacetime is not spherically symmetric, then only the solution (1) is recovered. However, a new possibility appears when the spacetime is spherically symmetric in the traditional sense, but not invariant with respect to G. Then the 3-spaces t = const are the same as in the Robertson-Walker models, but the sign of the spatial curvature, k, may depend on time:

$$ds^2 = D^2(t,r)\,dt^2 - R^2(t)\{dr^2/[1 - k(t)r^2] + r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)\}$$

Thus it is in principle possible here to have an initially closed Universe ($k > 0$) evolve into an open one ($k < 0$). The field equations admit solutions of this kind when the fluid source is not moving along the t-lines and the pressure is different from zero. It would be interesting to investigate the geometry and physics of such a spacetime in the course of opening up, but unfortunately the single equation to which the Einstein field equations reduce in this case is so involved that no solution has been found explicitly. Only an intuitive argument may be given in favor of the models with the changing sign of spatial curvature. The behavior of the closed and the open Robertson-Walker Universes is explained in Newtonian terms as follows: the Universe is closed when its mass-density relative to the rate of expansion is large enough to halt the expansion by gravitational interaction, and is open otherwise. Therefore, to allow for the change of an initially closed Universe into an open one, a mechanism for the decay of mass would have to operate. In principle, one such mechanism is conceivable. As is well known, the pressure of a gas or a fluid, which in Newtonian physics can only tend to expand the volume of the medium, gives in the general relativity theory a positive contribution to the energy-density: a pressure term appears in the equation of equilibrium, which must be balanced by an increased gradient of pressure, just as if pressure had its weight. Consequently, at large values, the pressure exhibits the opposite tendency to enhance the self-gravitation of the medium. It is therefore conceivable that initially the contribution of pressure to self-gravitation is large enough to close the Universe, while in the latter stages of expansion this effect becomes negligible, and the Universe opens up. In agreement with the equations obtained, this should be possible only with nonzero pressure.

This kind of solutions was overlooked during the investigations of the classical Bianchi-type homogeneous models just because there one demands the transitive group of the 3-spaces to be a symmetry group (or subgroup) of the whole spacetime, and then indeed k = const necessarily [6].

A more detailed account of this subject will be published elsewhere [4].

References:

[1] N. Salie, private communication.

[2] C. B. Collins, Gen. Rel. Grav. 10, 925, (1979).

[3] A. Krasiński, M. Perkowski, submitted to Gen. Rel. Grav. See also this volume.

[4] A. Krasiński, submitted to Gen. Rel. Grav.

[5] H. Nariai, Sci. Rep. Tôhoku Univ. 34, 160 (1950), 35, 62 (1951).

[6] M. P. Ryan, L. C. Shepley, Homogeneous relativistic cosmologies. Princeton University Press, Princeton 1975, pp. 104 and 147.

Note added in print: The solution reported here as a new one was already found by H. Stephani, Commun. Math. Phys. 4, 137 (1967).