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ELLIPSOIDAL SPACETIMES

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In newtonian hydrodynamics the only case when a rotating finite fluid body may be treated analytically is when the body is of ellipsoidal shape. Therefore one should not hope that any more general situation might be described in general relativity by exact solutions of the field equations. Looking for a relativistic analog of the newtonian theory of ellipsoidal figures of equilibrium one must however define what he means by an ellipsoid in a curved space.

Let us consider the euclidean space filled with a congruence of concentric and coaxial ellipsoids of revolution. Let us connect with this congruence the coordinate system (r, ϑ, ϕ) such that $x = g(r)\sin\vartheta\cos\phi$, $y = g(r)\sin\vartheta\sin\phi$, $z = r\cos\vartheta$, where the value of r is equal to the semiaxis of the ellipsoid lying along the symmetry axis, and the value of $g(r)$ for each r is the other semiaxis. Such coordinates are orthogonal only when $g^2 - r^2 = \text{const}$. Let us choose this simplest case for consideration. When $g^2 - r^2 = a^2 > 0$, the ellipsoids are oblate. Then the metric form of the euclidean space assumes the form:

$$ds^2 = \frac{r^2 + a^2\cos^2\vartheta}{r^2 + a^2}\, dr^2 + (r^2 + a^2\cos^2\vartheta)\, d\vartheta^2 + (r^2 + a^2)\sin^2\vartheta\, d\phi^2 \qquad (1)$$

The surface $r = \text{const}$ describes the surface of an ellipsoid, and this definition is independent of the space in which the ellipsoid is imbedded.

Now, in a curved 3-space containig a congruence of ellipsoids the subspaces $r = \text{const}$ should be the same as in (1). A reasonable guess is then to set the coefficient of dr^2 equal to an unknown function $f^2(r,\vartheta)$. The explicit dependence of f on r and ϑ should be decided from geometric considerations.

Let us now proceed to 4-dimensional spacetime. Here the apparent shape of a surface depends in general on the family of observers performing the description of the surface. Consequently, if a congruence of ellipsoids is to exist in a curved spacetime, its members will be described as ellipsoids by a limited family of observers only. It is most likely that, if there is matter in the spacetime, then the local rest spaces of matter will be appropriate for considering the ellipsoidal figures of equilibrium.

Let u^α be the velocity field of matter, and let $g_{\alpha\beta}$ be the metric tensor of the spacetime. Then the 3-tensor $(g_{\alpha\beta} - u_\alpha u_\beta)$ should be identified with the metric form of the curved 3-space filled with ellipsoids. If, moreover, we assume that the whole spacetime is stationary and axisymmetric in addition to being "ellipsoidal" then its metric will be:

$$ds^2 = [U^{-1}(1 - kV)dt + kd\varphi]^2 - f^2dr^2 - (r^2 + a^2\cos^2\vartheta)\, d\vartheta^2 - (r^2 + a^2)\sin^2\vartheta[d\varphi - (V/U)dt]^2$$

where $k(r,\vartheta)$ is an arbitrary function, and $U(r,\vartheta)$ and $V(r,\vartheta)$ are the u^0 and u^φ components of the four-velocity of matter.

In the above definition it was crucial to have matter in the spacetime, to be able to define the 3-spaces splitting into ellipsoids. Now, we may take a tentative definition that an empty spacetime will be called ellipsoidal when there exists in it any congruence of observers whose local rest-spaces have the metric form given by (1) with f^2 inserted before dr^2. It may be shown very easily that both the Kerr and Kerr-Newman solutions are ellipsoidal in this second sense. This might suggest that they are connected with gravitational fields of bodies of ellipsoidal shape, and it may help to decide finally the problem of existence of a material source for the Kerr metric. All the suggestions made up to now that the Kerr metric has no other source than a black hole are merely guesses based on the fact that it proved to be difficult to find a source. This is however not a serious foundation for such a strong claim.